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An iterated local search algorithm for the team orienteering problem with variable profits

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ABSTRACT

The orienteering problem (OP) is a routing problem that has numerous applications in various domains such as logistics and tourism. The objective is to determine a subset of vertices to visit for a vehicle so that the total collected score is maximized and a given time budget is not exceeded. The extensive application of the OP has led to many different variants, including the team orienteering problem (TOP) and the team orienteering problem with time windows. The TOP extends the OP by considering multiple vehicles. In this article, the team orienteering problem with variable profits (TOPVP) is studied. The main characteristic of the TOPVP is that the amount of score collected from a visited vertex depends on the duration of stay on that vertex. A mathematical programming model for the TOPVP is first presented and an algorithm based on iterated local search (ILS) that is able to solve modified benchmark instances is then proposed. It is concluded that ILS produces solutions which are comparable to those obtained by the commercial solver CPLEX for smaller instances. For the larger instances, ILS obtains good-quality solutions that have significantly better objective value than those found by CPLEX under reasonable computational times.

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1. Introduction

The orienteering problem (OP) is a multi-level optimization problem that has numerous applications in various domains such as logistics (Golden, Wang, and Liu 1987) and tourism (Souffriau *et al.* 2008). Vansteenwegen, Souffriau, and Van Oudheusden (2011) define the OP as a combination of vertex selection and determining the shortest Hamiltonian path between the selected vertices. The main objective is to determine a path, limited by the total travel time or distance, which visits some vertices in order to maximize the collected score from the visited vertices.

The team orienteering problem (TOP) is an extension of the OP with multiple paths. Each path is limited by the total travel time. The objective is to maximize the total collected score from all paths. Some recent work related to the TOP can be found in Dang, Guibadj, and Moukrim (2013), Ferreira, Quintas, and Oliveira (2014) and Ke *et al.* (2015). There are many different variants of the OP to accommodate additional aspects of real-world problems, such as multiple vehicles, vehicle capacities, customer time windows, stochastic/time-dependent travel times, and stochastic and variable profits. Vansteenwegen, Souffriau, and Van Oudheusden (2011) provide a comprehensive survey on the OP and its variants up to 2009. Gunawan, Lau, and Vansteenwegen (2016) extend the survey by focusing on more recent work on the OP and its latest variants, such as the capacitated OP and generalized OP.

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One recent variant of the OP is the orienteering problem with variable profits (OPVP), as presented by Erdoğan and Laporte (2013). In the OPVP, a visit at a particular vertex can be extended to collect more scores. To replicate such a situation, Erdoğan and Laporte (2013) introduce discrete passes, where each pass on a particular vertex represents a constant time incurred. The more passes the visit made, the longer the duration of stay.

In this article, a new variant of the OPVP, namely the team orienteering problem with variable profits (TOPVP), is introduced. In the context of logistics applications, a path can be based on a vehicle that needs to visit a certain number of vertices, and profits from vertices are considered as collected scores. The TOPVP extends the OPVP by considering multiple paths or vehicles. Therefore, the total collected scores from all paths is the main objective of the TOPVP.

Applications of the OPVP as described by Erdoğan and Laporte (2013) can be relevant for TOPVP as well. As an example, multiple boats may be considered when planning fishing routes where the amount of fish caught in each location is dependent on the time spent there. Another potential application of the TOPVP is in the deployment of multiple military units to different conflict zones for peacekeeping missions, whereby the peacekeeping organization strives to maximize the stability in the region and is still able to cease operations within the stipulated timeline. The TOPVP can be applied to organizing humanitarian logistics, whereby the amount of humanitarian aid received in each location is proportional to the time spent there. It can also be used to model the tourist trip design problem, where tourists may prefer to stay longer at a particular location or attraction.

A mathematical programming model for the TOPVP, which would be solved by the commercial solver CPLEX, is first introduced. Owing to the limitation of using this solver to solve large instances, a heuristic based on iterated local search (ILS) to solve the TOPVP is then proposed. It is concluded that the proposed algorithm performs well with short computational times for solving large TOPVP instances.

The remainder of this article is organized as follows. In Section 2, a literature review of the OP including its variants is provided. The problem description and the mathematical programming model for the TOPVP are then given in Section 3. In Section 4, the proposed algorithm is described. Section 5 reports numerical experiments that are performed on modified benchmark instances. Finally, in Section 6, the main achievements and possible future works are summarized.

2. Literature review

Tsiligirides (1984) was the first to define the standard OP. Important assumptions in the OP include perfect knowledge of the score specified for each vertex and the time incurred for the edges. In addition, each vertex can be visited only once, except for the start and the end vertices, which are commonly referring to the same vertex. In this article, it is assumed that the start and end vertices are the same vertex as well.

One characteristic of the classical OP is that the duration of staying at any vertex during a visit is fixed; therefore, the full score or profit is collected upon reaching the vertex. However, in certain situations, especially those related to logistics problems, the time spent in a particular vertex for a vehicle to unload the delivery has to be determined. This problem is referred to as the OPVP (Erdoğan and Laporte 2013).

In the OPVP, the vehicle is permitted to prolong the duration of stay. In this case, each vertex is assigned a profit that can potentially be collected, with the actual collected amount depending on the time spent on the vertex. The vehicle is not compelled to collect the full profit. Erdoğan and Laporte (2013) introduced discrete passes to represent the vehicle's duration of stay at a particular vertex for the discrete model of the OPVP. More specifically, making a pass means staying for a predefined amount of time at a vertex. The amount of time spent on a vertex is additive according to the number of passes made. The profit collected over the duration of stay is described using the growth or decay function, which determines the rate of increase or decrease of profit collected per pass made, respectively. The rest of the conditions for the OPVP remain identical to those of the OP. Hence, making multiple passes on a vertex does not equate to visiting the vertex multiple times since each vertex can be visited only once.

A unified branch-and-cut algorithm for OPVP has been proposed as the solution approach, using adapted inequalities from the covering tour problem formulation (Gendreau, Laporte, and Semet 1997). Since no prior research has been done on this, there were no benchmark instances available for the OPVP. As such, Erdoğan and Laporte (2013) modified test instances from the TSPLIB, which is a library of sample instances for the travelling salesman problem (TSP), to generate OPVP test instances. Even though optimality can be achieved when solving most instances, excessive computational times are required to solve the larger instances.

Since there is limited literature available for the OPVP, reviewing the solution approaches to the TOP may provide deeper insights into the development of heuristics for TOPVP. This is because the TOP and TOPVP share largely similar characteristics, with the exception of the variable profit component. Chao, Golden, and Wasil (1996b) proposed a heuristic for the TOP that involves two phases: initialization and improvement phases. In the initialization phase, a feasible solution is constructed using the farthest vertices from the start vertex. Additional paths involving non-visited vertices in the initial feasible solution are constructed in this phase as well. The improvement phase consists of iterating the sequence of two-point exchange, one-point movement and 2-Opt operators until terminating conditions are met. Archetti, Hertz, and Speranza (2007) proposed four comparable metaheuristics that are variants of tabu search and variable neighbourhood search heuristics. The metaheuristics first generate an initial feasible solution using the initialization phase of Chao, Golden, and Wasil's (1996b) heuristic.

Some researchers have proposed exact algorithms to solve the TOP, as described next. Boussier, Feillet, and Gendreau (2007) proposed a branch-and-price algorithm using column generation to solve the relaxed master problem, and then using the branch-and-bound method to obtain an integer solution to the TOP. Keshtkaran *et al.* (2016) proposed two algorithms to solve the TOP: a branch-and-price approach and a branch-and-cut-and-price approach, while El-Hajj, Dang, and Moukrim (2016) introduced a cutting plane algorithm to solve the TOP.

Archetti, Hertz, and Speranza's (2007) proposed metaheuristics are one of the leading algorithms for the TOP in terms of achieving the best known solution, the average gap to the best known solution and the average computational time. In addition, these high-performance algorithms are able to construct feasible paths for non-included vertices, allow infeasible solutions during the search procedure, as well as alternate between operators that increase objective value and decrease travel time.

3. Team orienteering problem with variable profits

3.1. Problem description

The TOPVP can be described by an undirected graph $G = (V, E)$, where $V = \{0, 1, \dots, |V|\}$ is the set of vertices and E is the set of edges. Vertices 1 to $|V|$ are potential vertices to visit, whereas vertex 0 corresponds to the start and end vertices of the paths. Each vertex $i \in V$ is designed with a score S_i as well as an associated collection parameter $\alpha_i \in [0, 1]$. The amount of score collected at each vertex i depends on the duration of stay at that vertex and its collection parameter α_i. The duration of stay at vertices is represented by discrete passes. Each pass made at vertex i incurs a constant time cost r_i. The collection parameter is used to model the decay of the collected score, where each pass made at a vertex allows collection of $100\,\alpha_i\%$ of the remaining score.

A travel time t_{ij} is associated with every edge $(i, j) \in E$. Thus, the total travel time on a particular path is contributed to by the travel time across edges as well as the number of passes made at visited vertices. The objective of the TOPVP is to determine a set of paths P such that the collected score by all paths is maximized. The amount of time required to traverse between two vertices (i, j) is assumed to be symmetrical, *i.e.* $t_{ij} = t_{ji}$. In addition, the travel time associated with every edge satisfies the triangle inequality.

Standard constraints applied to the OP (Vansteenwegen, Souffriau, and Van Oudheusden 2011) are also applied to the TOPVP, such as each vertex can be visited at most only once, except for the start vertex, which is the same as the end vertex; each path has to start and end at the start and end vertices, respectively; and each path is limited by the time budget T.

3.2. Mathematical programming model

The formulation of the TOPVP is extended from the OPVP discrete model (Erdoğan and Laporte 2013). The theoretical maximum number of passes at vertex i is denoted as m_i, where $m_i \leq \lfloor (T - 2t_{0i})/r_i \rfloor$. Below is the list of decision variables for the proposed mathematical programming model:

x_{ijp} = 1, if a visit to node i is followed by a visit to node j on path p; 0, otherwise

y_{ilp} = 1, if l or more passes are performed at vertex i on path p; 0, otherwise

$$\text{Maximize} \sum_{i \in V \backslash \{0\}} S_i \sum_{l \in \{1,\ldots,m_i\}} \alpha_i (1-\alpha_i)^{l-1} y_{ilp} \tag{1}$$

subject to:

$$\sum_{j:(0,j)\in E, p \in P} x_{0jp} = \sum_{i:(i,0)\in E, p\in P} x_{i0p} = |P| \tag{2}$$

$$\sum_{p \in P} y_{i1p} \leq 1, (i \in V\backslash\{0\}) \tag{3}$$

$$\sum_{i:(i,k)\in E, i\neq k} x_{ikp} = \sum_{j:(k,j)\in E, j\neq k} x_{kjp} = y_{k1p}, (k \in V\backslash\{0\}, p \in P) \tag{4}$$

$$y_{ilp} \leq y_{i,l-1,p}, (i \in V\backslash\{0\}, l \in \{2,\ldots,m_i\}, p \in P) \tag{5}$$

$$y_{i(m_i+1)p} = 0, (i \in V\backslash\{0\}, p \in P) \tag{6}$$

$$\sum_{(i,j)\in E, i\neq j} t_{ij} x_{ijp} + \sum_{i\in V} r_i \sum_{l\in\{1,\ldots,m_i\}} y_{ilp} \leq T, (p \in P) \tag{7}$$

$$2 \leq u_{ip} \leq |V|, (i \in V\backslash\{0\}, p \in P) \tag{8}$$

$$u_{ip} - u_{jp} + 1 \leq (|V|-1)(1-x_{ijp}), (i,j \in V\backslash\{0\}, p \in P) \tag{9}$$

$$y_{01p} = 0, (p \in P) \tag{10}$$

$$y_{ilp} = 0 \text{ or } 1, (i \in V\backslash\{0\}, l \in \{1,\ldots,m_i\}, p \in P) \tag{11}$$

$$x_{ijp} = 0 \text{ or } 1, ((i,j) \in E, p \in P) \tag{12}$$

The objective function (1) maximizes the total collected score/profit from visited vertices of all paths. It is worth noting that the total profit collected from each vertex will then be $S_i \sum_{l\in\{1,\ldots,m_i\}} \alpha_i (1-\alpha_i)^{l-1} y_{ilp}$, which resembles a finite geometric series. Constraints (2) designate vertex 0 as the start and end vertices for each path. Constraints (3) ensure that across all paths, each vertex can be visited at most only once, with the exception of vertex 0. Constraints (4) ensure the connectivity between the edges and the vertex. In other words, each vertex that is visited must be the origin and the destination for a pair of edges.

Constraints (5) ensure that in order to make further passes at a particular vertex, the preceding pass must be made. Constraints (6) ensure that the paths do not exceed the maximum allowable passes of the visited vertices. If the maximum allowable passes of all vertices are not limited by exogenous reasons, then constraints (6) can be relaxed. Constraints (7) ensure that the total time allocated does not exceed the time budget T for every path. Since both the edge costs and time incurred from making passes at vertices are subtracted from the total time allocated, costs and time are treated as synonymous in this article. Constraints (8) and (9) prevent the formation of subtours. These constraints are adopted from those proposed by Divsalar, Vansteenwegen, and Cattrysse (2013), Palomo-Martinez *et al.* (2017) and Vansteenwegen *et al.* (2009). Constraints (10) ensure that no passes are made at vertex 0. Constraints (11) and (12) are the integrality constraints.

Erdoğan and Laporte (2013) noted that in the case where $a_i = 1$ for all $i \in V \backslash \{0\}$, the OPVP is reduced to a selective travelling salesman problem (STSP) (Laporte and Martello 1990). Since the STSP is NP hard and is a special case of the OPVP, by extension, the TOPVP is also NP hard. This implies that an exact solution algorithm may be beyond computational reach and that attempting to obtain a suboptimal solution through heuristics will be more appropriate.

4. Proposed iterated local search algorithm

In this section, an algorithm to solve the TOPVP, which is mainly based on ILS, is proposed. The details of this proposed ILS are described below.

The proposed algorithm is an extension of the ILS proposed by Chao, Golden, and Wasil (1996a), as shown in Figure 1. In this subsection, an overview of the algorithm structure will first be presented, followed by more detailed elaboration of the different operators used in the algorithm.

4.1. Overview of the proposed algorithm

ILS is defined as a local search method that iteratively applies a local search to perturbations of the current locally optimal solution (Lourenço, Martin, and Stützle 2003). The four basic requirements

```
Step 1. Initialization
        Perform initialization
        Set record = objective value of the initial solution
        Set deviation = 5% × record
Step 2. 1st Improvement Phase
        For k = 1, 2, 3, …                                        (K loop)
            For l = 1, 2, 3, …                                    (L loop)
                Perform two-point exchange
                Perform one-point exchange
                Perform 2-opt
                If no adjustment has been made, end L loop
                If a solution with higher objective value is obtained, then
                    Set record = objective value of new solution
                    Set deviation = 5% × record
            End L loop
            If record is not updated for 5 consecutive iterations, then
                Go to Step 3
            Else
                Perform re-initialization 1 (free k vertices)
        End K loop
Step 3. Re-initialization 2
        Perform re-initialization 2 (free k vertices, k is the stopping value in K loop)
        Set deviation = 2.5% × record
Step 4. 2nd Improvement Phase
          Same sequence as Step 2 except deviation is now set to 2.5% × record
```

Figure 1. Iterated local search.

of the ILS are: (1) an initial solution; (2) a perturbation guideline to deconstruct the locally optimal solution; (3) a local search to seek improvements in the solution; and (4) an acceptance criterion to determine which solution the local search is continued from.

Similar to Chao, Golden, and Wasil's (1996a) algorithm, the proposed ILS consists of an initialization, two improvement steps and two re-initialization steps. The initialization step constructs the initial feasible solution. The two improvement steps represent the local search methods, seeking possible improvements in the current solution. The two re-initialization steps perturb the locally optimal solution for the next iteration of improvement.

The acceptance criterion used in ILS is based on an optimization algorithm called the record-to-record travel (RRT) (Dueck 1993). In the RRT, the best solution obtained thus far is set as the *record.* Any configuration found that is an improvement over *record* is set as the new *record*. The proposed ILS attempts to seek further improvement using the new configuration. In addition, a constant percentage of the *record* is set as the acceptance threshold called *deviation*. Whenever the algorithm fails to find a configuration that is an improvement, the best configuration that deteriorates the solution within the *deviation* will be chosen to be worked on.

The sequence of steps to be executed for ILS is as follows:

Step 1 (initialization phase): An initial solution is constructed using the initialization process, which is discussed in Section 4.3. The objective value of the initial solution is set as the *record*, while the *deviation* is set at 5% of the *record*.

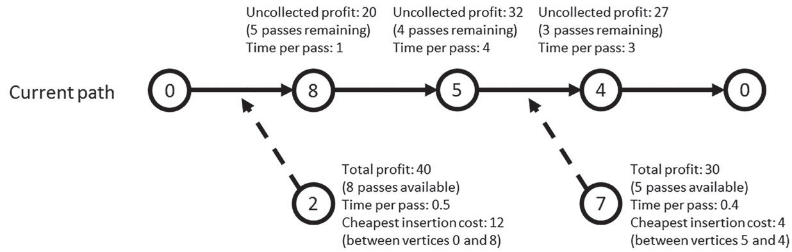
Step 2 (improvement phase): The improvement phase consists of two loops: the inner loop will be referred to as L loop while the outer loop will be referred to as K loop. In the L loop, a local search for improvement is conducted. This local search is a sequence of operators consisting of two-point exchange, one-point movement and 2-Opt operators. These operators will be discussed in Section 4.4. At the end of the sequence, if a solution with a higher objective value is obtained, then *record* and *deviation* are updated. The local search is iterated until no exchange or movement of vertices can be performed, ending the L loop. Note that it is possible to have no improvement in the objective function value after performing exchanges or movements. In the K loop, re-initialization 1 (described in Section 4.5) is performed to perturb the solution obtained from the L loop. The new solution will then be iterated through the L loop again. Subsequent performing of re-initialization 1 perturbs the solution according to how many times re-initialization 1 has been performed. A global variable k is used to track the number of re-initialization 1 steps performed. The terminating condition for the K loop is when no new *record* is achieved for five consecutive iterations.

Step 3 (re-initialization 2): Re-initialization 2 (described in Section 4.5) is performed using the last k value of Step 2. The *deviation* is also set to 2.5% of the current *record*.

Step 4 (improvement phase 2): The second improvement phase follows the same sequence of events as Step 2 (first improvement phase) with the exception of the *deviation* used in RRT exchanges and movements.

4.2. Insertion criteria

Before elaborating on the different operators used in the TOPVP algorithm, there is a need to introduce a newly conceived criterion, called the *most efficient* criterion. This criterion is used extensively in the proposed algorithm to accommodate the discrete pass component of the TOPVP that is used to represent the duration of stay at a vertex. Owing to the discrete passes, the construction heuristic used in the ILS algorithm is required to decide between increasing the passes made on the vertices currently in a path of interest or inserting an additional vertex that is not in that path. Thus, the commonly used construction heuristics, such as the cheapest insertion and nearest neighbour heuristics, cannot be directly applied since there is no comparison between the increments of passes and the additional insertion of vertices. To address this issue, the *most efficient* criterion designates a score to



Vertex	Profit Available / Remaining	Total Time Required for the Passes	Cheapest Insertion Cost	Score
8	20	5	-	4
5	32	16	-	2
4	27	9	-	3
2	40	4	12	2.5
7	**30**	**2**	**4**	**5**

Figure 2. Illustration of the '*most efficient*' criterion.

each vertex using the profit available from the vertex and the total time required to collect that profit, as shown below:

$$\text{Score} = \frac{\text{Profit available for collection}}{\text{Total time required to collect the profit}} \tag{13}$$

Considering a path of interest, all vertices can be categorized as (1) vertices in the path or (2) unassigned vertices. For any vertex i that is in the path, the profit available refers to the remaining profit that is uncollected and calculated using $p_i - p_i \sum_{l\in\{1,\ldots,m_i\}} \alpha_i(1-\alpha_i)^{l-1} y_{ilp}$. The total time required to collect this profit is then $r_i(m_i - \sum_{l\in\{1,\ldots,m_i\}} y_{ilp})$, which is the product of the remaining allowable passes and the duration of a single pass. In short, the score for vertices assigned to the path is the uncollected profit per unit of time required.

On the other hand, the insertion cost for vertices not assigned to the path is an additional consideration. In this case, the profit available refers to the entire profit of the vertex, p_i. The total time required then consists of the cheapest insertion cost and the time incurred to collect the entire profit. Let vertex k be the vertex not assigned to the path and vertices i and j be consecutive vertices in the path. Thus, the expression for the total time required is $r_k m_k + \min_{k\neq i,j}(t_{ik} + t_{kj} - t_{ij})$, where $r_k m_k$ is the time required to collect the entire profit at vertex k and $\min_{k\neq i,j}(t_{ik} + t_{kj} - t_{ij})$ is the cheapest insertion cost. After calculating the score for every vertex, the vertex with the highest score can either be inserted into the path or have an additional pass made.

Figure 2 depicts the process of using the *most efficient* criterion. Let the current path be 0–8–5–4–0, and vertices 2 and 7 be unassigned; the score for every vertex is then calculated according to the *most efficient* criterion. Consequently, vertex 7 scored the highest and is inserted between vertices 5 and 4.

4.3. Initialization phase

In the initialization step, vertices that cannot be theoretically visited given the time budget are first removed. In other words, any vertex i for which $2t_{i0} + r_i > T$ is removed from consideration by the heuristic. This is because the vehicles have to return to the depot and any vehicle that visits these vertices will always violate the time budget allocated owing to the triangle inequality assumption. The vertices that are not removed in this process are referred to as feasible vertices.

The next process in the initialization step will be to construct paths using the feasible vertices. Similarly to Chao, Golden, and Wasil's (1996a) and Archetti, Hertz, and Speranza's (2007) heuristics, ILS constructs additional feasible paths that are not in the solution such that every feasible vertex is assigned to a path. The $|P|$ paths with the highest profit collected constitute the solution and will be referred to as the set of paths P_{TOPVP}. Thus, the sum of the profit collected from each path in P_{TOPVP} is the objective value. The set of the remaining paths will be referred to as P_{NTOPVP}. It is possible for a path in P_{NTOPVP} to replace another path in P_{TOPVP} as long as the profit collected is higher.

First, C vertices are chosen as candidate vertices, where C is defined as min($|P| + 1$, number of feasible vertices). These candidate vertices are selected to be the vertices that are farthest away from the depot in terms of the time taken to travel to the vertex to make a single pass, $t_{0i} + r_i$. Subsequently, out of the C vertices, $|P|$ vertices are chosen and inserted as the first vertices to each of the $|P|$ paths. All remaining vertices that are yet to be assigned are then inserted into the $|P|$ paths using the *most efficient* criterion until the paths are full. Paths are considered full when no additional vertices can be inserted or no additional passes can be made.

If all $|P|$ paths are full and there are vertices left unassigned, then additional paths are constructed until all feasible vertices are assigned. These additional paths are constructed using the same idea of constructing $|P|$ paths. The $|P|$ paths with the highest profit collected are in the set P_{TOPVP}, while the remaining paths are in the set P_{NTOPVP}.

In the case when $C > |P|$, since only $|P|$ vertices are chosen out of C vertices, there are $\begin{pmatrix} C \\ |P| \end{pmatrix}$ possible combinations of vertices chosen. For ILS, each of the combinations will be initialized according to the above description. The combination with the highest objective value will then be set as the initial solution. Subsequently, the *record* and *deviation* can be obtained. This will then conclude the initialization phase of the ILS algorithm.

When determining the value of C, it is possible that $C \leq |P|$. In this case, obtaining the optimal solution is trivial. This is because the number of vertices that are feasible is less than or equal to the number of paths. Therefore, the optimal solution is to assign each vertex to the different paths randomly, making the maximum allowable passes m_i to each vertex and ensuring the feasibility of solutions.

4.4. Improvement phase

Using the initial solution generated in the initialization phase, the solution is then improved by performing three different operators of ILS.

4.4.1. Two-point exchange

The objective of the two-point exchange is to seek possible improvement in the solution by exchanging vertices from the paths in P_{TOPVP} with vertices from the paths in P_{NTOPVP}. Since there is no shared vertex allocated in both P_{TOPVP} and P_{NTOPVP}, the complexity involved is $\mathrm{O}(|V|^2)$.

More specifically, all the vertices in P_{TOPVP} are checked for exchanges one at a time. When considering the exchanges, the passes made at the two involved vertices are first set to one. The vertices are then inserted into their respective paths using the cheapest insertion criterion with complexity $\mathrm{O}(|V|)$. Finally, the passes made at the two involved vertices are gradually increased until the paths are full, which would only cost $\mathrm{O}(1)$ for the process. Therefore, the overall complexity of this operator would be $\mathrm{O}(|V|^2) \times [\mathrm{O}(|V|) + \mathrm{O}(1)] = \mathrm{O}(|V|^3)$.

Any exchange that causes the path in P_{TOPVP} to become infeasible will not be considered; exchanges that cause the path in P_{NTOPVP} to become infeasible are still acceptable. Upon discovering an exchange that will increase the objective value, whether due to an increase in profit collected by the path in P_{TOPVP} or due to the path in P_{NTOPVP} replacing one of the paths in P_{TOPVP}, this exchange is performed immediately. Whenever an exchange is successful, the two-point exchange will proceed to the next vertex in P_{TOPVP}. In the case where the path in P_{NTOPVP} becomes infeasible owing to the exchange, an additional path containing only the depot and the responsible vertex will be constructed.

It may be possible for a vertex in P_{TOPVP} not to have any exchanges with any vertex in P_{NTOPVP} that will result in an improvement in objective value. In this case, the RRT acceptance criterion will be utilized. Exchanges that will result in a small decrease in the objective value are now considered. The amount of decrease, however, must be within the *deviation*. If such RRT exchanges are available, then the RRT exchange with the highest objective value will be performed. In other words, the best RRT exchange that is within *deviation* will be performed only if the vertex in P_{TOPVP} has no exchanges with any vertex in P_{NTOPVP} that will improve the objective value. If no feasible or RRT exchange is within *deviation*, then no exchange will be performed for that vertex in P_{TOPVP}.

At the end of the two-point exchange, every path in P_{NTOPVP} is deconstructed and the vertices are unassigned. New paths are then constructed using the *most efficient* criterion to reassign all the vertices that were unassigned previously. This is because during the exchanges, the paths containing only one vertex and the depot may be constructed whenever the path in P_{NTOPVP} becomes infeasible during an exchange. These ineffective paths are eliminated by reconstructing all the paths in P_{NTOPVP}. During the reconstructing process, it is possible for a path in P_{NTOPVP} to replace a path in P_{TOPVP} owing to the higher profit collected leading to an improvement in objective value. Note that the paths are always kept feasible throughout the two-point exchange.

4.4.2. One-point movement

One-point movement is the operator used after two-point exchange in the local search component. One-point movement attempts to improve the solution by relocating vertices from one path to another. In particular, every feasible vertex is checked for possible movement, one at a time. When considering a possible movement for a candidate vertex from its original path to a designated path, both paths will have the number of passes made at every vertex set to one. The candidate vertex is then inserted into the designated path using the cheapest insertion heuristic. The process of inserting a vertex and evaluating the insertion to other paths involves $\mathrm{O}(|V|^2)$ complexity. Finally, after the movement is made, the passes made for the vertices in both paths are increased using the *most efficient* criterion until the paths are full. This process involves $\mathrm{O}(|V|)$ complexity. Therefore, the total complexity of this operator is the addition of $\mathrm{O}(|V|^2)$ and $\mathrm{O}(|V|)$ complexities, and is thus $\mathrm{O}(|V|^2)$.

In addition, when selecting the designated path for insertion, paths with higher profit collected are given greater priority. Any movement that will cause either of the paths to become infeasible will not be considered. Upon discovering a movement that improves the objective value, the candidate vertex is relocated immediately. The one-point movement will then proceed to evaluate the next candidate vertex.

In addition, it is possible for a candidate vertex not to have any movement that will improve the objective value. Similarly to the two-point exchange, the RRT acceptance criterion becomes active. In this case, movements that do not affect or cause a small decrease in the objective value are considered; the amount of decrease must be within the *deviation*. If RRT movements are possible, then the movement with the highest objective value will be performed.

Regardless of the type of movement made, a path in P_{NTOPVP} with higher profit collected may replace another path in P_{TOPVP}. The paths in P_{TOPVP} and P_{NTOPVP} are sorted whenever a movement is made. Finally, when every vertex has been evaluated for one-point movement, any empty path is removed.

4.4.3. 2-Opt

The 2-Opt technique is used after two-point exchange and one-point movement have been completed in order to reduce the total edge cost incurred by the paths in P_{TOPVP} and P_{NTOPVP}. In doing so, there may be opportunities for more exchanges and movements in later iterations. There should be no improvement in the objective value due to 2-Opt. The operator is applied to each path in P_{TOPVP} and P_{NTOPVP} with the complexity of $O(|V|^2)$.

Note that, as mentioned earlier in the overview of the structure of the proposed algorithm, the two parameters *record* and *deviation* are updated only after a sequence of two-point exchange, one-point movement and 2-Opt is completed. If no new *record* is found after five consecutive iterations, then the terminating condition for the corresponding improvement phase has been achieved. Otherwise, the solution obtained will be perturbed using re-initialization 1 as described below.

4.5. Re-initialization phase

The two different re-initialization phases are described as follows.

4.5.1. Re-initialization 1

To avoid being restricted to a particular neighbourhood, re-initialization 1 is used to prepare the solution for the next iteration of the local search. It is the perturbation component of ILS. In this step, vertices with the lowest profit collected are removed from each of the paths in P_{TOPVP}. The number of vertices removed from each path is determined by a variable k. As the iteration count for the local search increases, the value of k is increased by one unit. In other words, more vertices are removed from the paths in P_{TOPVP} in subsequent re-initialization 1 steps. New paths containing the removed vertices are then constructed using the *most efficient* criterion. Finally, the new P_{TOPVP} is determined and the next iteration of the local search will be performed on this new configuration.

4.5.2. Re-initialization 2

In re-initialization 2, the vertices are removed differently from re-initialization 1. In particular, instead of removing vertices with the lowest profit collected, vertices with the smallest ratio of profit collected to insertion cost are removed from each of the paths in P_{TOPVP}. The number of vertices removed from each path is the stopping value of k for re-initialization 1 in improvement phase 1. Note that throughout the ILS algorithm, re-initialization 2 will be performed only once.

New paths containing the removed vertices are then constructed using the *most efficient* criterion. Finally, the new P_{TOPVP} is determined and improvement phase 2 will begin. In addition, the threshold of RRT exchanges and movements is reduced so as to only perturb the solution slightly during improvement phase 2. In improvement phase 2, the *deviation* is reduced to 2.5% of the *record*.

5. Computational experiments and results

In this section, the results obtained after applying the proposed algorithm to test instances for the TOPVP are presented. The algorithm is coded using Visual Studio 2013 in C++ and executed on computers with an Intel Core i5-4570 central processing unit (CPU) at 3.20 GHz and 8 GB RAM.

5.1. Benchmark instances

Since no benchmark TOPVP instances are readily available, there is a need to generate the test instances to evaluate the proposed algorithm. As such, the scheme proposed by Erdoğan and Laporte (2013) is used to generate benchmark OPVP instances using the TSP test instances.

The TSP instances used are kroA100, kroB100, kroC100, kroA200 and kroB200, which are obtained from the TSPLIB. The number in the instance's name represents the number of vertices. The first vertex from the data file is set as the depot. The edge costs t_{ij} are determined from the vertex coordinates using Euclidean distance. Note that the 200-vertex instances are extensions of the 100-vertex instances, with the first 100 vertices being the same.

The OPVP instances are then used to generate the TOPVP instances. This is done by taking the one-path OPVP and dividing the time budget by the number of paths in TOPVP (Chao, Golden, and

Table 1. Formulae used to generate parameters for team orienteering problem with variable profits (TOPVP) instances.

$X_{max}X_{min}Y_{max}Y_{min}$	=	Maximum X coordinate, minimum X coordinate, maximum Y coordinate and minimum Y coordinate of the vertices considered for the problem
p_i	=	$10 + (X_i + Y_i \text{mod} 90)$
a_i	=	$(10 + (((X_i + Y_i \text{mod} 20)/20) \times 90))/100$
r_{max}	=	$(X_{max} - X_{min} + Y_{max} - Y_{min})/50$
r_{min}	=	$(X_{max} - X_{min} + Y_{max} - Y_{min})/100$
r_i	=	$r_{min} + (((X_i + Y_i \text{mod} 15)/15) \times (r_{max} - r_{min}))$
T	=	$(2.5 \times (X_{max} - X_{min} + Y_{max} - Y_{min}))/\lvert P\rvert$

Wasil 1996a). In other words, the accumulative total time budget for the paths in TOPVP is the same as the time budget for OPVP. Table 1 summarizes the formulae used to generate the parameters from the vertex coordinates.

Verification and validation of the ILS algorithm were conducted using the CPLEX solver to solve for the optimal solution for small test instances. The objective is to compare the optimal solution of the instances with that of ILS. The optimality gap for the ILS solutions can be determined, although this is only for small test instances.

The factors tested in the verification and validation experiments are the number of vertices $|V|$, number of paths $|P|$ and time budget for each path T. Since the test instances from TSPLIB are in sets of 100 vertices (kroA100, kroB100 and kroC100) and 200 vertices (kroA200 and kroB200), solving to optimality using all the test instances with such significant sizes will be beyond computational reach. As such, to reduce the size of the experiments, only the initial 15 vertices from each test instance are used, and experiments for five, 10 and 15 vertices are conducted. In addition, the number of paths

Table 2. Validation results for kroA100.

Test instance	$\lvert P\rvert$	$\lvert V\rvert$	T	CPLEX objective value	ILS objective value	Optimality gap (%)	CPLEX CPU time (s)	ILS CPU time (s)
kroA100	1	5	5000	81.00	81.00	0.000	4.59	0.22
	1	5	7500	173.07	169.13	2.277	3.85	0.28
	1	5	10000	261.91	258.79	1.191	4.52	0.36
	1	5	12500	279.24	279.02	0.079	4.54	0.37
	1	10	5000	251.99	250.50	0.591	8.81	0.31
	1	10	7500	294.19	293.07	0.381	7.82	0.57
	1	10	10000	394.15	391.31	0.721	10.50	0.66
	1	10	12500	500.66	491.68	1.794	9.50	0.72
	1	15	5000	249.90	249.35	0.220	14.37	0.32
	1	15	7500	334.64	330.11	1.354	19.90	0.37
	1	15	10000	455.30	450.27	1.105	38.28	0.41
	1	15	12500	589.74	579.64	1.713	22.10	0.50
	2	5	5000	154.98	154.98	0.000	3.98	0.24
	2	5	7000	269.09	268.18	0.338	4.06	0.24
	2	5	9000	279.82	279.77	0.018	4.56	0.40
	2	10	5000	346.97	345.48	0.429	10.28	0.42
	2	10	7000	464.52	464.03	0.105	10.80	0.62
	2	10	9000	541.10	538.50	0.481	16.86	0.93
	2	15	5000	383.47	381.02	0.639	72.62	0.69
	2	15	7000	572.56	570.26	0.402	227.90	1.09
	2	15	9000	715.09	691.35	3.320	188.76	0.91
	3	5	5000	195.61	195.61	0.000	4.54	0.28
	3	5	7000	279.45	279.45	0.000	4.73	0.54
	3	5	9000	280.00	280.00	0.000	4.88	0.44
	3	10	5000	422.76	416.79	1.412	16.24	0.63
	3	10	7000	547.06	539.97	1.296	12.20	0.71
	3	10	9000	553.80	553.17	0.114	50.72	0.49
	3	15	5000	476.36	473.91	0.514	219.76	0.62
	3	15	7000	732.18	713.49	2.553	800.44	0.74
	3	15	9000	788.10	779.49	1.099	>1000	1.07

Note: ILS = iterated local search; CPU = central processing unit.

$|P|$ tested for each test instance is one, two and three. The solver CPLEX 12.6.2 was used to solve the TOPVP on a computer with an Intel Xeon E5-1603 CPU at 2.80 GHz and 16 GB RAM.

The objective value of the solutions obtained from CPLEX and ILS are recorded in Tables 2 and 3. Part of the verification process involves checking whether the objective values from ILS have exceeded the objective value of the optimal solution and, as can be observed, these objective values do not exceed the optimal objective value. In addition, the solutions are checked to ensure that the paths remained feasible. Furthermore, the maximum optimality gap for all the experiments conducted is 3.32%, which is an acceptable value for small instances.

For the time budget assigned for each path/vehicle, arbitrary values are picked for the verification and validation experiments. This is because by evaluating ILS over a comprehensive range of time budget values, the verification and validation process can be more credible. To illustrate, the edge costs calculated from the test instances can go up to about 3000 time units. For experiments with only five vertices considered or 5000 time units allocated for each vehicle, the optimality gap is likely to be 0%. This can be attributed to the elimination of vertices that require more than 2500 time units when visiting from the depot. These vertices are eliminated from consideration since they cannot be visited without exceeding the time budget. As such, the reduction in the number of vertices considered resulted in ILS being more likely to converge to the optimal solution.

In contrast, for larger experiments with more than 7000 time units allocated, each vertex can be visited. In this case, the performance of ILS drops slightly, and optimality is achieved in some experiments only. In addition, the largest time budget allocated for each vehicle was set at 12500 and 9000 time units for the experiments with a single vehicle and multiple vehicles, respectively. This is

Table 3. Validation results for kroB100.

Test instance	$\|P\|$	$\|V\|$	T	CPLEX objective value	ILS objective value	Optimality gap (%)	CPLEX CPU time (s)	ILS CPU time (s)
kroB100	1	5	5000	103.00	103.00	0.000	3.92	0.27
	1	5	7500	242.40	241.54	0.355	4.45	0.28
	1	5	10000	269.80	269.80	0.000	4.63	0.28
	1	5	12500	270.00	270.00	0.000	4.62	0.30
	1	10	5000	283.99	283.99	0.000	8.81	0.33
	1	10	7500	423.79	421.99	0.425	9.64	0.47
	1	10	10000	533.99	533.82	0.032	10.83	0.46
	1	10	12500	560.70	560.66	0.007	10.53	0.57
	1	15	5000	286.42	286.37	0.017	14.21	0.43
	1	15	7500	446.82	446.41	0.092	57.53	0.60
	1	15	10000	635.85	630.50	0.841	26.13	0.75
	1	15	12500	703.17	702.68	0.070	19.42	0.73
	2	5	5000	178.00	178.00	0.000	3.82	0.24
	2	5	7000	270.00	270.00	0.000	4.56	0.47
	2	5	9000	270.00	270.00	0.000	4.70	0.47
	2	10	5000	435.61	435.61	0.000	10.28	0.47
	2	10	7000	560.31	560.07	0.043	13.85	0.43
	2	10	9000	561.00	561.00	0.000	10.11	0.84
	2	15	5000	455.52	450.09	1.192	22.95	0.41
	2	15	7000	662.38	650.67	1.768	582.18	0.93
	2	15	9000	733.95	733.87	0.011	225.33	1.02
	3	5	5000	178.00	178.00	0.000	4.76	0.27
	3	5	7000	270.00	270.00	0.000	4.56	0.43
	3	5	9000	270.00	270.00	0.000	4.62	0.50
	3	10	5000	468.61	468.60	0.002	11.22	0.64
	3	10	7000	561.00	561.00	0.000	78.81	0.46
	3	10	9000	561.00	561.00	0.000	9.64	0.67
	3	15	5000	523.68	511.81	2.267	224.13	0.61
	3	15	7000	733.42	733.36	0.008	>1000	0.66
	3	15	9000	734.99	734.98	0.003	>1000	0.68

Note: ILS = iterated local search; CPU = central processing unit.

because given a larger time budget, the paths in the solution will be longer and the maximum allowable passes made at each path will be higher. Thus, setting a larger time budget is likely to be beyond computational reach.

5.2. Computational results

In this section, experiments were conducted for the full range of vertices using the test instances kroA100, kroB100, kroC100, kroA200 and kroB200. Results obtained from CPLEX after 1000 s of computational time were compared with the results obtained by ILS. The objective is to evaluate

Table 4. Definitions of column headings.

CPLEX upper bound :	Best upper bound obtained by CPLEX at 1000 s
CPLEX best solution :	Objective value of the best solution obtained by CPLEX at 1000 s
TOPVP objective value :	Objective value of the ILS
TOPVP CPU time (s) :	Computational time required for the ILS
Upper bound–TOPVP gap (%) :	Percentage deviation of the ILS's objective value from CPLEX upper bound

Note: TOPVP = team orienteering problem with variable profits; ILS = iterated local search; CPU = central processing unit.

Table 5. Computational results for kroA100, kroB100 and kroC100.

Test instance	$\lvert P\rvert$	$\lvert V\rvert$	T	CPLEX upper bound	CPLEX best solution	ILS objective value	ILS CPU time (s)	Upper bound–ILS gap (%)
kroA100	2	25	7020	874.39	762.40	736.49	0.66	15.77
	2	50	7186	1464.12	868.92	1205.71	1.50	17.65
	2	75	7240	1783.85	1005.67	1372.54	1.75	23.06
	2	100	7351	2026.70	1408.40	1539.49	4.51	24.04
	3	25	4680	777.44	649.45	667.23	0.86	14.18
	3	50	4791	1288.17	921.60	967.35	1.12	24.91
	3	75	4827	1625.96	916.86	1174.92	2.07	27.74
	3	100	4901	1881.80	1150.23	1318.98	2.17	29.91
	4	25	3510	746.92	588.89	580.05	0.74	22.34
	4	50	3593	1143.35	761.17	757.50	0.64	33.75
	4	75	3620	1486.59	829.77	956.83	1.77	35.64
	4	100	3673	1701.83	982.12	1083.80	2.64	36.32
kroB100	2	25	6718	1057.95	702.81	795.07	0.75	24.85
	2	50	7051	1550.25	1122.16	1129.98	1.44	27.11
	2	75	7275	1934.81	1189.21	1410.98	2.34	27.07
	2	100	7391	2519.20	1548.81	1837.54	5.79	27.06
	3	25	4478	589.34	573.72	570.74	0.85	3.16
	3	50	4701	1216.24	1042.29	1017.78	1.49	16.32
	3	75	4850	1707.65	1285.83	1344.68	1.03	21.26
	3	100	4927	2319.70	1559.52	1731.71	1.97	25.35
	4	25	3359	520.67	520.67	519.50	0.79	0.22
	4	50	3526	1137.17	951.78	942.26	1.47	17.14
	4	75	3638	1605.68	1076.38	1193.96	1.33	25.64
	4	100	3696	2252.69	1207.40	1514.31	1.85	32.78
kroC100	2	25	7020	832.62	735.28	689.65	0.71	17.17
	2	50	7186	1418.87	1008.78	1118.80	1.63	21.15
	2	75	7240	1930.83	1238.18	1368.79	2.56	29.11
	2	100	7345	2285.38	1240.78	1665.63	3.90	27.12
	3	25	4680	794.99	627.43	586.49	0.69	26.23
	3	50	4791	1244.62	953.44	973.19	1.73	21.81
	3	75	4827	1762.39	1114.87	1233.01	2.48	30.04
	3	100	4897	2151.57	1157.68	1497.63	2.73	30.39
	4	25	3510	557.57	517.37	491.55	0.83	11.84
	4	50	3593	1081.96	828.61	818.23	1.58	24.38
	4	75	3620	1547.31	951.18	1029.44	2.40	33.47
	4	100	3673	1941.39	1158.59	1225.85	2.48	36.86

Note: ILS = iterated local search; CPU = central processing unit.

Table 6. Computational results for kroA200 and kroB200.

Test instance	$\lvert P \rvert$	$\lvert V \rvert$	T	CPLEX upper bound	CPLEX best solution	ILS objective value	ILS CPU time (s)	Upper bound–ILS gap (%)
kroA200	2	125	7345	2556.54	1126.84	1755.72	3.90	31.32
	2	150	7380	2826.82	1267.49	1914.66	7.17	32.27
	2	175	7380	2994.77	466.22	2036.53	5.21	32.00
	2	200	7380	3117.18	155.99	2111.68	8.94	32.26
	3	125	4897	2362.61	1091.55	1687.18	3.34	28.59
	3	150	4920	2638.86	1072.10	1722.68	4.77	34.72
	3	175	4920	2790.47	858.58	1860.15	4.89	33.34
	3	200	4920	2906.92	247.89	1917.95	4.97	34.02
	4	125	3673	2263.87	1136.59	1405.30	2.17	37.92
	4	150	3690	2388.89	1081.03	1498.14	2.94	37.29
	4	175	3690	2538.22	1235.60	1595.99	3.18	37.12
	4	200	3690	2700.82	734.89	1630.20	3.70	39.64
kroB200	2	125	7391	2727.64	1091.29	1905.82	5.19	30.13
	2	150	7391	2957.56	1357.81	2048.07	6.83	30.75
	2	175	7391	3102.08	1319.55	2119.59	11.95	31.67
	2	200	7401	3320.49	948.71	2233.36	8.82	32.74
	3	125	4927	2475.03	1530.28	1802.51	4.08	27.17
	3	150	4927	2675.39	1496.86	1920.98	3.74	28.20
	3	175	4927	2833.15	1329.10	2102.80	5.33	25.78
	3	200	4934	3093.34	617.07	2116.00	7.36	31.59
	4	125	3696	2393.64	1271.46	1605.69	2.54	32.92
	4	150	3696	2617.37	1486.56	1813.00	4.63	30.73
	4	175	3696	2828.17	1298.20	1853.34	3.67	34.47
	4	200	3701	3001.25	897.02	1976.09	3.67	34.16

Note: ILS = iterated local search; CPU = central processing unit.

the performance of the ILS algorithm using the best upper bound as well as the best feasible integer solution obtained by CPLEX.

The numbers of vertices used for kroA100, kroB100 and kroC100 instances are varied at 25, 50, 75 and 100, using the initial vertices for each instance. The numbers of vertices used for kroA200 and kroB200 instances are varied at 125, 150, 175 and 200 to avoid repetition of computational results due to using the same initial vertices for each instance. Experiments are conducted for two-, three- and four-path TOPVPs. The definitions of the column headings are given in Table 4. The results for the kroA100, kroB100 and kroC100 instances are presented in Table 5. Table 6 presents the results for the larger instances kroA200 and kroB200.

As observed from Tables 5 and 6, the ILS algorithm is able to obtain a solution using considerably smaller amounts of computational time, even for the large instances kroA200 and kroB200. The maximum computational time recorded is 11.95 s for the two-path 175 vertices TOPVP using the test instance kroB200. This is due to the ILS algorithm using operators that are not computationally intensive. In addition, the computational time for experiments with comparatively more paths is generally lower. This can be attributed to the time budget allocated for each path being lower than in experiments with fewer paths. Since the time budget is lower, the paths constructed are shorter and the maximum number of passes allowed for each vertex is lower. Thus, the possibility of achieving improvement from the local search component of ILS is lower. Hence, ILS is able to converge to a solution using fewer iterations in the improvement phases.

Although the gap between the solution obtained from ILS and the upper bound is considerably large, it can be reasonably justified. Given the termination time being set at 1000 s, there is already a large optimality gap between the upper bound and the solution obtained by CPLEX. Hence, the actual optimality gap for ILS is likely to be smaller than the gap reported in Tables 5 and 6. This is seen in the kroB100 instance with $|V| = 4$, $|P| = 25$ and $T = 3359$, where the optimal solution was obtained by CPLEX and the optimality gap for ILS is only 0.22%.

Even though the average gap reported might be considerably large, ILS manages to produce near-optimal solutions for a few experiments. More specifically, for kroB100 three- and four-path instances, the gaps for the experiments using only 25 vertices are considerably small, 3.16% and 0.22%, respectively. The reason for this is two-fold: the number of vertices is only 25, and the time budgets allocated for the three- and four-path experiments are relatively low. As such, the number of feasible vertices in the time budget constraint is small, enabling ILS to reach a near-optimal solution. On the other hand, for experiments with more vertices available, ILS is unable to achieve a comparable small gap. By the same argument, the number of feasible vertices for ILS to consider is large, which explains the much larger gap reported.

Given the considerably large gap between ILS and the upper bound, improving the local search as well as the perturbation is certainly a probable notion. The current operators used are relatively simple. For example, two-point exchange is essentially the swapping of two vertices, while one-point exchange attempts to insert an additional vertex into another path. Hence, the use of more sophisticated operators for the local search may lead to possible improvements that these simple operators are unable to achieve. Also, the perturbation component can be made more rigorous, as modifying the solution further after a local search may improve the chances of discovering a better local optimal solution. Furthermore, the current computational time of ILS is considerably small. As such, implementing these suggestions should cause the computational time to increase only slightly.

6. Conclusion

A variant of the OP, namely the TOPVP is introduced. For the TOPVP, multiple paths are involved in collecting scores which are dependent on the time spent at the visited vertices. In this article, the TOPVP is formulated as a mathematical programming model that could be extended to other models. One of the future plans for extending the model is to include the correlated effect among vertices (in the context of the tourist trip design problem). The objective function for this extended model will consist of two components: the total of the collected scores from visited nodes and a quadratic score function that captures spatial correlations among nodes (Yu, Schwager, and Rus 2014).

An ILS algorithm has been developed to solve the modified benchmark TOPVP instances. The results obtained from solving some modified benchmark instances by ILS are compared with those obtained by the CPLEX solver, which is able to solve small instances to optimality. For these small instances ranging up to 15 vertices, ILS is able to achieve optimality in several experiments using considerably shorter computational time. ILS is then applied to larger instances ranging up to 200 vertices. While the computational time needed for ILS is low, the gap between the ILS results and the upper bound obtained by CPLEX solver after 1000 s is still considerably small, and the ILS is able to obtain better solutions than the CPLEX solver after 1000 s in most of the larger instances. However, the development of a more effective heuristic incorporating more advanced operators to achieve a smaller optimality gap is a possible direction for future research.

In the current TOPVP model, it is assumed that split deliveries are not allowed. Archetti *et al.* (2014) introduced the split delivery capacitated team orienteering problem. When split deliveries are allowed, the number of vehicles used may be reduced. It is theoretically shown that split deliveries may increase the profit twice compared with the constraint that limits each customer to be served by at most one vehicle. It is also shown experimentally that the profit increase due to split deliveries varies according to the instance. As part of future work, the proposed algorithm could be extended to tackle this problem. Other applications of the OP, such as the vehicle routing problem and the tourist trip design problem, which have similar characteristics to the TOPVP can also be considered. Finally, the idea of using stronger mathematical programming models and exact algorithms, such as the branch-and-cut approach in Erdoğan and Laporte (2013), to generate better upper bound values will be considered for future work as well.

Disclosure statement

No potential conflict of interest was reported by the authors.

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