A Model for Fresh Produce Shelf-Space Allocation and Inventory Management with Freshness-Condition-Dependent Demand

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A significant amount of work has investigated inventory-control problems associated with fresh produce. Much of this work has considered deteriorating inventory control with many models having been proposed for various situations. However, no researchers have specifically studied fresh produce, which has its own special characteristics. Most research categorizes fresh produce into more general deteriorating categories with random lifetimes and non-decaying utilities. However, this classification is not reasonable or practical because the freshness of an item usually plays an important role in influencing the demand for the produce. In this paper, a single-period inventory and shelf-space-allocation model is proposed for fresh produce. These items usually have a very short lifetime. The demand rate is assumed to be deterministic and dependent on both the displayed inventory (the number of facings of items on the shelves) and the items' freshness condition (which decreases over time). Several problem instances of different sizes are provided and solved by a modified generalized reduced gradient algorithm.

Key words: inventory; shelf-space allocation; fresh produce; optimization

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1. Scope and Purpose

The profit on general foods, such as canned goods, frozen vegetables, fruit juice, etc., is gradually decreasing due to highly competitive retail conditions. The demand for these products is also slowing. On the other hand, the demand for some other merchandise, such as fresh produce, organic food, and children clothes, has increased dramatically due to improving living standards. This requires retailers to concentrate more in these areas (Johnson 2002). We formulate a model to assist in ordering and shelf-allocation decisions for the retail of fresh produce, such as vegetables, fruits, fresh meats, etc. We draw together two distinct areas; a packing problem (Bennell and Dowsland 2001; Burke et al. 2004, 2006) and inventory control (Nahmias 1982, Raafat 1991, Goyal and Giri 2001).

The main characteristics of the items we consider are their very short shelf-life and decaying utilities over time. Preliminary experiments are conducted (using a modified generalized reduced gradient algorithm), on our proposed model, to demonstrate that good solutions can be found. Most of the literature has treated fresh produce as deteriorating items with a random lifetime and non-decaying utilities (Nahmias 1982, Goyal and Giri 2001). We assume that the produce has a continuous utility and physically deteriorates over time. Freshness is one of the main criteria to evaluate a product’s quality and could dramatically affect its demand if its condition is inferior. To obtain good financial performance from fresh goods requires adoption of strict temperature control and intelligent inventory and shelf-management systems. Furthermore, although many deteriorating inventory models have been proposed, most of them are based on the analysis of a single item excluding the constraints of shelf space that arise when considering a range of goods. No researchers, to our knowledge, have yet integrated a deteriorating-inventory model with a shelf-space-allocation model (which plays a very important role in retail decision making due to the scarce shelf-space resources). We formulate a fresh-produce-management model that can simultaneously decide the ordering policy as well as allocate shelf space among different items, together with consideration of utility (i.e. freshness) deterioration.

2. Background

Perishable inventory has been intensively studied and many models have been proposed. See Nahmias (1982), Raafat (1991), and Goyal and Giri (2001) for comprehensive reviews. However, most models assume that a fixed fraction of the inventory deteriorates over time but the utilities of the items do not decay before their expiration dates. Few models specifically consider fresh produce with the characteristics we mentioned in Section 1. Some of the shortcomings of previous models include: 1). Most models (Liu 1990, Jain and Silver 1994) assume that fresh produce has a random lifetime (usually assuming an exponentially distributed lifetime) but the item utilities do not decay over time. Hence different ages of items capture the same demand however fresh they are as long as they are not completely spoiled. This is contradictory to the common-sense view that freshness is one of the most important quality criteria for fresh produce. 2). Some models (Mandal and Phaujdar 1989, Giri et al. 1996) formulate the demand as a deterministic function of instantaneous inventory with the assumption that all stock could be displayed on the shelves. However, this situation seldom occurs because the shelf space for fresh food is normally limited. It is also expensive due to the low temperature requirements. Therefore, only a part of the inventory can be displayed on the shelf. Shelf-space allocation among different items is especially important in this situation. The significance of shelf-space allocation for non-perishable merchandise has been addressed in (Kotzan and Evanson 1969, Curhan 1972, Borin et al. 1994, Urban 1998, Yang and Chen 1999, Bai and Kendall 2005). 3). The approaches that were used to optimize the models (Ben-Daya and Raouf 1993, Kar et al. 2001) disregarded the integer nature of the solution and assumed that the objective function is a quasi-concave function and is differentiable. The last assumption is usually too strict for problems involving many constraints.

The literature has classified different deteriorating inventory models into two types: fixed-lifetime models and random-lifetime models. Examples of fixed-lifetime models include photographic films, medicine, computer chips, and canned food. A major characteristic of this type of model is that inventory allows for different ages of items with either a first-in-first-Out (FIFO) or last-in-first-out (LIFO) issuing policy (Nandakumar and Morton 1990, Liu and Lian 1999). However, fresh produce is usually treated as a typical example of a random-lifetime product due to the uncertain spoilage (Liu 1990, Jain and Silver 1994). These models usually assumed a constant fraction of inventory decay over time (called *exponential decay* in some publications).

Since fresh produce has only a very limited shelf life, most of the literature employed a single-period inventory model although different forms of demand function are used. Both stochastic and deterministic demand inventory models were proposed for the perishable products. Ben-Daya and Raouf (1993) proposed a multi-item, single-period perishable-inventory model with a uniform distribution for demand. The objective was to maximize the total profit of all the items during one period. The "optimal" solution was calculated by a Lagrangian optimization with the assumption that the objective is differentiable. The integer nature of the variables was also disregarded. Furthermore, the method is not efficient when there are many constraints. Rajan et al. (1992) proposed a dynamic pricing and ordering decision-making model for decaying produce, in which the demand was assumed to be deterministic and dependent on the selling price. The products are assumed to have an exponential deterioration. Abad (1996) formulated the demand function as a function of instantaneous price. A closed-form mathematical procedure was carried out to solve the problem and parameter sensitivities were analyzed. However, the approach is heavily dependent on the mathematical description of the model so that adding even a single constraint could invalidate this approach. Some other models formulated the demand as a deterministic function of instantaneous inventory. Mandal and Phaujdar (1989) formulated a single-period inventory model for deteriorating items. The demand rate was linearly dependent on the instantaneous inventory level, and the inventory deteriorated according to a given function. Backordering was allowed, and holding and shortage costs were considered. The objective was to minimize the average cost. The variables included the time slots for different inventory stages and maximal stock level and maximal stock deficit. Giri et al. (1996) formulated the demand as a polynomial function of the instantaneous inventory in their perishable-inventory model, which also assumed an exponential decay. The objective is to maximize the profit, with order quantity and reorder point (or cycle time) as decision variables. Some time-dependent demand functions were also proposed in deteriorating inventory models to capture changing demand over time. Xu and Wang (1990) assumed a linear time-dependent demand function within a limited time horizon. Exponentially time-dependent demand was also proposed to simulate a rapidly increasing/declining market (Hollier and Mak 1983, Zhou et al. 2003). Urban and Baker (1997) used a multiplicative demand function of price, time, and inventory level in their single-period inventory model with the aim of finding optimal ordering and pricing policies for non-perishable products.

The first research to consider the effect of utility deterioration on demand was Fujiwara and Perera (1993) in the formulation of an economic order quantity (EOQ) perishable inventory model. An exponential penalty function $a(e^{bt}-1)$ $(a>0, b>0)$ was used to measure the cost of keeping an aging item in inventory. A closed form of economic order quantities was obtained by a quadratic approximation of exponential terms. The results show that this model is consistent with other EOQ models with exponential decay. Sarker et al. (1997) also attempted to incorporate the negative effect of aging inventory on demand. In their production-inventory model, the demand function in the inventory build-up phase and depletion phase considered a constant term and a negative term that is proportional to the instantaneous inventory (i.e. $f(t) = D - bI(t)$, where $f(t)$ is the demand function, $b > 0$, D is constant demand, and $I(t)$ is the instantaneous inventory level). However, illogically, the demand during the inventory-depletion phase is actually an increasing function due to the continuous decrease of the inventory $I(t)$ over time. This contradicts the authors' initial intention to represent a declining demand with the aging of the inventory.

Almost all the models described above consider only a single item without any constraints, with the optimal solution usually obtained analytically. Recently, researchers have begun to incorporate shelf-space-allocation technologies into their inventory systems. Kar et al. (2001) proposed a single-period inventory model for multi-deteriorating items with constraints on shelf space and investment. The problem considers selling the deteriorating items from two stores. At the beginning of the period, the ordered items are separated into fresh items and items that have begun to deteriorate. The fresh items are shipped to the main store, selling at a high price, and the deteriorating items are delivered to the second store and sold at a lower price. During the period, all decayed items in the main store are retained and delivered to the second store. The demand rate in the first store was formulated as a function of the item's selling price and instantaneous inventory. However, the demand in the second store was dependent only on the selling price. A generalized reduced gradient (GRG) method was used to optimize the model. However, as stated in Lasdon et al. (1978), GRG may not be efficient or robust for larger problem sizes and can guarantee only a local optimum. Besides, the non-integer variables and continuous objective assumption are the major drawbacks of this approach in solving many NP-hard problems with integer variables. Hence, heuristic and meta-heuristic approaches (Glover and Kochenberger 2003, Burke and Kendall 2005) have been used in this area to optimize these models. Borin et al. (1994) used a simulated-annealing approach to solve a product-assortment and shelf-space-allocation problem. Genetic algorithms were employed in Urban (1998) to solve an integrated product-assortment, inventory, and shelf-space-allocation model.

3. Model Formulation

Instead of assuming that fresh food has a random lifetime with an exponential decay, we assume that fresh food has predictable expiration dates but their freshness condition also decreases continuously according to a known function over time. The demand for the fresh produce is deterministic and is dependent on both the displayed inventory level and their freshness condition. The main difference between these two assumptions is that the former assumes that all items that have not yet deteriorated capture the same demand however fresh they are. This may sound reasonable for long-life perishable items (like photographic films and medicine) but is unrealistic for fresh produce as freshness is one of the most important aspects in evaluating their quality. In this paper, all fresh items are assumed to have a fixed, but very short, lifetime and will not entirely lose their utilities before their expiration date. However, freshness keeps deceasing over time, which has an effect on demand. The assumption of a fixed lifetime of fresh produce, with decreasing utilities, is realistic considering the advances in food planting, packing, and conservation technologies, especially the introduction of temperature-control systems in most supermarkets.

We use the following notation:

$D_i(t)$	The demand function of item *i* over time.
$f_i(t)$	A decreasing function (within range [0,1]) representing the freshness condition of item *i* over time.
a_i	Scale parameter of item *i*.
$ß_i$	Space elasticity of item *i*.
s_i	Decaying rate of item *i*.
$I_i(t)$	Inventory level of item *i* at time *t*.
q_i	Procurement quantity of item *i*.
s_i	The number of the facings assigned to item *i*.
r_i	The surplus of item *i* at the end of the cycle.
W	Total shelf space available.
a_i	Shelf space required for one facing of item *i*.
p_i	Unit selling price of item *i*.

p_{di}	Unit discounted price of item i. This price should be low enough such that all of the remaining items at the end of period can be sold out in a very short time at this price.
c_{ai}	Unit acquisition cost of item i (or unit procurement price).
c_{hi}	Unit holding cost of item i (including the costs caused by inventory loses, damage, maintenance, interest, insurance, etc.).
c_s	Shelf cost per unit space.
C_{oi}	Constant order cost of item i (independent of the order quantity).
T_{ei}	Lifetime of item i after which the item is rotten (i.e. cannot be sold).
L_i	Lower bound of the number of facings of item i.
U_i	Upper bound of the number of facings of item i.
T_i	Length of the cycle period of item i.
HC_{1i}	Total holding cost during $[0, t_{1i}]$.
HC_{2i}	Total holding cost during $[t_{1i}, T_i]$.

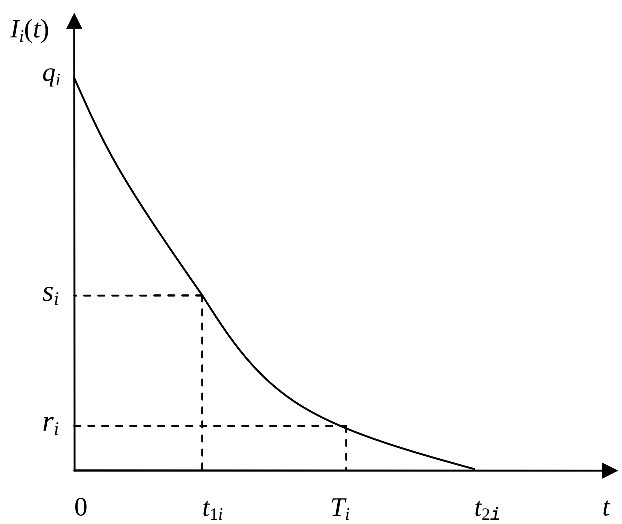

Figure 1: Graphical Representation of Inventory-Level Changes Over Time

Many researchers (e.g. Kar et al. 2001, Urban 2002) use the function in Figure 1 to describe the change of inventory level over time t. From time 0 to t_{1i}, s_i facings of item i are displayed on the shelf with some of the stock stored in the backroom. As sales are made, the items in the back-room are moved to the shelf until the stock in backroom reaches zero (corresponding to the point when time reaches t_{1i}). Therefore, during this period, the shelf is fully stocked and the demand is a function only of product freshness. From time t_{1i} to t_{2i}, the shelf is only partly stocked and the demand is dependent on both the freshness and the instantaneous inventory level. Once the time reaches point T_i, a new order of quantity q_i is placed for item i (assuming no lead time) and the r_i surplus of item i is removed from the shelf and is assumed to be sold at a discount price, p_{di}, immediately. We adopt this representation together with a polynomial demand function that is widely used in many shelf-space-allocation models (Corstjens and Doyle 1981, Giri et al. 1996, Urban and Baker 1997, Urban 1998):

$$D_i^*(t) = \begin{cases} a_i s_i^{b_i} & 0 \le t \le t_{1i} \\ a_i [I_i(t)]^{b_i} & t_{1i} < t \le t_{2i} \end{cases}$$

where a_i and b_i are scale parameters and the space elasticity of item i, respectively, and $a_i > 0$, $0 < b_i < 1$. We assume that the demand function conforms to a multiplicative form of the instantaneous inventory and the item's freshness condition, i.e. $D_i(t) = D_i^*(t) f_i(t)$ where $f_i(t)$ is a continuously decreasing function over time and $0 \le f_i(t) \le 1$. $f_i(t)$ could be a linear, quadratic, or exponential function of time. During the beginning of the period, the items are fresh and the value of the freshness function is almost 1. The demand rate is affected only by the displayed inventory level. However, as time passes, $f_i(t)$ gradually decreases and the demand is scaled down according to how long an item has been kept in inventory. To be consistent with the exponential-decay assumption in the literature, we assume that an items' freshness condition decreases exponentially over time, i.e. $f_i(t) = e^{-s_i t}$, where $s_i > 0$ is a constant decay rate. Hence we have

$$D_i(t) = D_i^*(t) f_i(t) = \begin{cases} a_i s_i^{b_i} e^{-s_i t} & 0 \le t \le t_{1i} \\ a_i [I_i(t)]^{b_i} e^{-s_i t} & t_{1i} < t \le t_{2i} \end{cases}.$$

Based on the assumptions above, the inventory level of item i can be described by the differential equation $dI_i(t)/dt = -D_i(t)$. During time [0, t_{1i}], we have

$$dI_i(t)/dt = -a_i s_i^{b_i} e^{-s_i t}. \tag{1}$$

with the boundary conditions $I_i(0) = q_i$ and $I_i(t_{1i}) = s_i$. The solution of (1) is $I_i(t) = q_i + \frac{a_i s_i^{b_i}}{s_i}(e^{-s_i t} - 1)$ and $t_{1i} = -\frac{1}{s_i} \ln(1 - \frac{(q_i - s_i) s_i}{a_i s_i^{b_i}})$. During time [$t_{1i}$, t_{2i}], we have the differential equation

$$dI_i(t)/dt = -a_i[I_i(t)]^{b_i} e^{-s_i t} \tag{2}$$

with the boundary conditions $I_i(t_{1i}) = s_i$ and $I_i(t_{2i}) = 0$. The solution of (2) is

$$I_i(t) = [\frac{a_i(1-b_i)}{s_i} e^{-s_i t} + K_i]^{\frac{1}{(1-b_i)}}$$

and

$$t_{2i} = -\frac{1}{s_i} \ln(1 - \frac{(q_i - b_i(q_i - s_i))s_i}{a_i(1-b_i)s_i^{b_i}})$$

where $K_i = [q_i - b_i(q_i - s_i)]s_i^{-b_i} - m_i$ and $m_i = \dfrac{a_i(1-b_i)}{s_i}$.

In general, we have the inventory function

$$I_i(t) = \begin{cases} q_i + \dfrac{a_i s_i^{b_i}}{s_i}(e^{-s_i t} - 1) & 0 \le t \le t_{1i} \\ [m_i e^{-s_i t} + K_i]^{\frac{1}{(1-b_i)}} & t_{1i} < t \le t_{2i} \end{cases} . \tag{3}$$

The length of cycle period T_i ($I_i(T_i) = r_i$) is

$$T_i = -\frac{1}{s_i} \ln\left[\frac{1}{m_i}(r_i^{(1-b_i)} - K_i)\right].$$

The holding cost during $[0, t_{1i}]$ is

$$HC_{1i} = c_{hi}\int_0^{t_{1i}} (q_i + \frac{a_i s_i^{b_i}}{s_i}(e^{-s_i t} - 1))dt = c_{hi}[(q_i - \frac{a_i s_i^{b_i}}{s_i})t_{1i} + (1 - e^{-s_i t_{1i}})\frac{a_i s_i^{b_i}}{s_i^2}] .$$

The holding cost during $[t_{1i}, T_i]$ is

$$HC_{2i} = c_{hi}\int_{t_{1i}}^{T_i} ([m_i e^{-s_i t} + K_i]^{\frac{1}{(1-b_i)}})dt .$$

The approximate expression of HC_{2i} is given in the Appendix. However, this part is very small and we use a simpler approximation (using $(s_i + r_i)/2$ as an approximation of average inventory during $[t_{1i}, T_i]$)

$$HC_{2i} = c_{hi}(s_i + r_i)(T_i - t_{1i})/2 .$$

Therefore, the average profit of item *i* per unit time is the total income less any costs involved, divided by the time of the period

$$M_i = \frac{1}{T_i}[p_i(q_i - r_i) + p_{di}r_i - c_{ai}q_i - C_{oi} - HC_{1i} - HC_{2i}] - c_s s_i a_i .$$

The objective is to maximize the overall profit of all items during the unit time:

$$\max \quad \sum_{i=1}^{n} M_i(s_i, q_i, r_i) \tag{4}$$

$$\text{subject to} \quad \sum_{i=1}^{n} s_i a_i \le W \tag{5}$$

$$L_i \le s_i \le U_i \qquad i = 1, 2, \ldots, n \tag{6}$$

$$r_i \le s_i \le q_i \qquad i = 1, 2, \ldots, n \tag{7}$$

$$r_i < q_i \qquad i = 1, 2, \ldots, n \tag{8}$$

$$0 < T_i \le T_{ei} \qquad i = 1, 2, \ldots, n \tag{9}$$

$$s_i, q_i \in \{1, 2, 3, \ldots\} \qquad i = 1, 2, \ldots, n \tag{10}$$

$$r_i \in \{0, 1, 2, \ldots\} \qquad i = 1, 2, \ldots, n \tag{11}$$

The decision variables are shelf space, order quantity and the amount of surplus at the end of the cycle. Constraint (5) ensures that the total shelf space allocated to each item is no more than the total available shelf space. Constraint (6) makes sure that the space allocated to each item must be within an upper and a lower bound. Constraint (7) ensures that the order quantity of each item must be greater than the shelf displayed quantity, which itself should be greater than the number of surplus items. Constraint (9) ensures that the span of one cycle period must be less than the product-validity period. Constraints (10) and (11) ensure that the number of facings, order quantity, and the number of surplus items are integers. The model is a non-linear combinatorial optimization problem and is difficult to optimize via conventional approaches.

If we have *n* products, the total number of variables is 3*n*. From the model, we have the upper and lower bounds of variables r_i ($0 < r_i \le s_i$) and s_i ($L_i < s_i \le U_i$ and lower bound of q_i ($q_i \ge s_i$). The upper bound of q_i can be obtained from constraint (9). Since

$$T_i = -\frac{1}{s_i} \ln\left[\frac{1}{m_i}(r_i^{(1-b_i)} - K_i)\right] \le T_{ei} .$$

we have

$$q_i \le \frac{1}{(1-b_i)} r_i^{(1-b_i)} s_i^{b_i} + \frac{a_i}{s_i} s_i^{b_i} - \frac{b_i}{(1-b_i)} s_i - \frac{a_i}{s_i} e^{-s_i T_{ei}} s_i^{b_i} .$$

If $\lfloor x \rfloor$ represent the largest integer no greater than *x*, the upper bound of order quantity q_i^{ub} is

$$q_i^{ub} = \left\lfloor \frac{1}{(1-b_i)} r_i^{(1-b_i)} s_i^{b_i} + \frac{a_i}{s_i} s_i^{b_i} - \frac{b_i}{(1-b_i)} s_i - \frac{a_i}{s_i} e^{-s_i T_{ei}} s_i^{b_i} \right\rfloor .$$

An interesting implication of the model is that inventory depletes exponentially over time (see (3)), which is consistent with the exponential-decay models in the literature. In addition, when $s_i \to 0$, $e^{-s_i t} \to 1 - s_i t$, so the inventory function becomes the same polynomial function derived in Urban (2002).

4. Optimization of the Model

We use GRG algorithm to search for good solutions to the model (4-11). The underlying ideas behind the algorithm are described in Gabriele and Ragsdell (1977) and Lasdon et al. (1978). The algorithm has been shown to be efficient in solving non-linear programming problems with smooth objective functions, and its applications in optimizing the inventory and shelf-space-allocation model include Urban (1998) and Kar et al. (2001), with good results. The GRG algorithm is embedded in many spreadsheet software packages. The one we use is Solver, which is included in Microsoft Excel 2002. However, the GRG algorithm has two major drawbacks: 1). It can solve only continuous-variable models. Although the package included in Microsoft Excel 2002 can deal with integer variables, it takes too long for the search to converge (1800 seconds computation time is needed for a problem with only 6 items, running on a Pentium IV 1.8GHZ with 256MB RAM. For a problem with 18 products, the algorithm does not converge even after one hour). 2). GRG usually gives only a local optimum that is closest to the starting point. Some preliminary experiments showed that, if the starting point is not carefully chosen, GRG performs very badly. Thus, we used a multi-thread GRG algorithm together with a solution-repair heuristic. Each thread of the algorithm can be divided into three sub-procedures: initialization, GRG, and solution repair; shown in Figure 2.

To prevent GRG from getting stuck at a local optimum, *MaxIter* runs of GRG were executed using different initial states (solutions) and the best solution was output as the final solution. In this application, we set *MaxIter* = 5 after some preliminary experiments. The initialization sub-procedure was used to generate a set of diverse solutions that can be used by GRG. Note that, because GRG is efficient only when handling continuous variables, a relaxed model (ignoring integrality constraints (10) and (11) was input into the Excel Solver. Therefore, the solution output by GRG was not feasible. The solution-repair sub-procedure is used to recover feasibility of the solution and further improve it by using a simple local-search method described in Figure 2 (several other rounding heuristics were tried, and this one generally performs best across the five problem instances we tested). All results were averaged over ten runs on a Pentium IV 1.8GHZ CPU with 256MB RAM, running Microsoft Windows 2000 Professional Version 5.

```
Set MaxIter;
Set iter = 0;
Loop
    //Initialization sub-procedure
    For each item i (1 ≤ i ≤ n) set s_i = L_i, q_i = s_i, r_i = 0;
    Loop
        Select a random item j;
        s_j = s_j + 1;
    Until no more facings can be added without violating the space constraint (5);
    For each item i
        Increase q_i until no improvement can be obtained in the objective value;
        Increase r_i until no improvement can be obtained in the objective value;
    Output solution S_0(q_i, s_i, r_i)

    //GRG calling sub-procedure
    S'=Solver(S_0);

    //Solution repair sub-procedure
    Round every s_i, q_i, r_i (1 ≤ i ≤ n) in S' to their nearest integers
    While space constraint (5) is violated
        Rank the items by their unit space profit value M_i/(a_i s_i);
        Delete one facing of the item with the smallest unit space profit value (if this opera-
        tion causes a constraint violation, the next item in the ranking list is considered);
    If free shelf space > the size of the smallest item
        Loop
            Rank the items by their unit space profit value M_i/(a_i s_i);
            Add one facing of the item with the largest unit-space profit value (the next
            item in the ranking list is considered if the operation generates a constraint vio-
            lation);
        Until no more facings can be added without violating the space constraint (5);
    For each item i (1 ≤ i ≤ n)
        Increase/decrease q_i until no improvement can be obtained in the objective value;
        Increase/decrease r_i until no improvement can be obtained in the objective value;
    Remember the best solution (S_best) found so far;
    iter++;
Until iter = MaxIter;
Output S_best;
```

Figure 2: Pseudo Code of the Multi-start GRG Algorithm

5. A Numerical Example

To provide a better understanding of the model and the solution procedure described above, a numerical example with 6 items was generated (denoted by BORIN94/6). The problem scale parameters (a_i) and space elasticities ($ß_i$) are taken from Borin et al. (1994) and the other parameters are listed in Table 1. The GRG algorithm described in Section 4 was run 10 times with different initial random solutions. The algorithm consistently returned the same solution, which is shown in Table 2. For comparison, an exhaustive search was also carried out to get an optimal solution, also shown in Table 2. For this numerical example, the solution obtained by GRG is very close to the optimal solution. The relative deviation from optimality is only 0.04% ($\frac{347.58-347.45}{347.58}100\%$).

Table 1: Parameters of the Numerical Example

Item	a_i	p_i	c_{ai}	c_{hi}	p_{di}	C_o	a_i	$ß_i$	s_i
1	0.028	5.03	2.46	0.19	1.23	34.3	28.53	0.1532	0.06
2	0.061	9.37	5.67	0.20	2.84	48.9	23.62	0.2273	0.07
3	0.025	5.10	2.70	0.26	1.35	35.6	25.59	0.2089	0.06
4	0.060	11.48	6.11	0.16	3.06	47.9	22.40	0.2143	0.04
5	0.036	6.74	3.53	0.30	1.77	33.9	15.62	0.2955	0.03
6	0.033	5.97	3.41	0.27	1.71	39.1	10.50	0.3104	0.03

W=0.608(m^2), c_s=5.0(pounds/m^2/unit time), L_i =1, U_i =12, T_{ei} =7(days)

Table 2: Solution of the Numerical Example

	Solution by GRG				Optimal Solution			
Item	q_i	s_i	r_i	T_i	q_i	s_i	r_i	T_i
1	83	3	0	2.68	81	2	0	2.78
2	78	2	0	3.17	78	2	0	3.17
3	77	3	0	2.61	77	3	0	2.61
4	88	3	0	3.35	88	3	0	3.35
5	64	3	0	3.17	64	3	0	3.17
6	50	1	0	5.19	56	2	0	4.68
Objective	347.45				347.58			

6. Larger Problem Instances

Although numerical examples are helpful in understanding the model and testing the performance of the solution procedure, it is necessary to test the algorithm over larger problem instances. We created four benchmark problem instances using the parameters in Table 3. The problem sizes range from 18 to 64 products. These datasets can be downloaded from the Online Supplement to this paper on the journal's website. Here we provide the computational results of the modified GRG algorithm, shown in Table 4. The modified GRG algorithm is quite robust on the

Table 3: Parameters of Problem Instances

Parameters	**Values**	**Parameters**	**Values**
n	18/32/49/64	L_i	1
a_i	U(10, 30)	U_i	12
$ß_i$	U(0.15, 0.3)	p_{di}	$0.5c_{ai}$
s_i	U(0.03, 0.1)	c_s	5.0 pounds/m^2/day
a_i	U(0.01,0.09) m^2	C_o	U(30, 50) pounds
c_{ai}	$N(100a_i$, 0.4) pounds	T_{ei}	7 days
p_i	$N(1.8c_{ai}$, 0.4) pounds	W	2.5* *minSpace*
c_{hi}	U(0.1,0.3) pounds		

$U(a, b)$: uniform distribution $N(c, d)$: normal distribution
minSpace: the minimal space requirement to satisfy products' number-of-facings lower bounds

five tested problem instances. With BORIN94/6 and FRESH2, all ten runs consistently returned the same solution although each run started from different, random initial solutions. For the other three instances, the difference between the best solution and worst solution, among the ten runs, is very small and the standard deviations are less than 1, a small value compared with the objective values. All solutions obtained by the algorithm satisfy the integer constraints and are therefore feasible solutions.

Table 4: The Computational Results of the GRG Algorithm on Five Problem Instances

	BORIN94/6	FRESH2	FRESH3	FRESH4	FRESH5
n	6	18	32	49	64
av. obj.	347.45	1129.60	2056.46	3163.98	4387.16
best obj.	347.45	1129.60	2057.15	3164.59	4387.73
worst obj.	347.45	1129.60	2055.17	3163.33	4386.66
std. dev	0.00	0.00	0.97	0.51	0.43
av. cpu	3.2	73.6	74.3	179.2	209.7

av. obj.: the average objective value over 10 runs
best obj.: the best objective value over 10 runs
worst obj: the worst objective value over 10 runs
std. dev.: absolute standard deviation of 10 results obtained by GRG
av. cpu: average cpu time consumed by GRG (in seconds)

7. Conclusions

A single-period inventory and shelf-space-allocation model has been proposed for fresh produce. The demand is assumed to be deterministic and conforms to a multiplicative form of the displayed stock-level and items' freshness conditions. The items' freshness condition is assumed to drop exponentially over time but could still capture some demand. The model is consistent with deteriorating inventory models reported in literature, in which an exponential decay in the inventory is assumed. Unlike other work, the proposed model considers the integer nature of the solution. Five benchmark problem instances were generated for the fresh-produce inventory-control and shelf-space-allocation problem. A modified GRG algorithm was used to search for good solutions and their computational results were reported. The algorithm used in this paper ensures integrality of the decision variables.

Appendix

Denote $y(t) = [m_i e^{-s_i t} + K_i]^{\frac{1}{(1-b_i)}}$. Divide the range [$t_{1i}$, T_i] into k identical ranges by point $x_0 = t_{1i}$, x_1, $x_2, \ldots, x_k = T_i$. We have

$$HC_{2i} = c_{hi} \int_{t_{1i}}^{T_i} ([m_i e^{-s_i t} + K_i]^{\frac{1}{(1-b_i)}}) dt \approx \frac{c_{hi}(T_i - t_{1i})}{k} [\frac{1}{2}(y(x_0) + y(x_k)) + y(x_1) + \ldots + y(x_{k-1})]$$

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